Magnetostatic Shield Analysis by Double Layer Charge Formulation Using Difference Field Concept

K. Ryu[1], T. Yoshioka[1], S. Wakao[1], Y. Takahashi[2], K. Ishibashi[3] and K. Fujiwara[2]

[1]Department of Electrical Engineering and Bioscience, Waseda University, Tokyo 169-8555, Japan
[2]Department of Electrical Engineering, Doshisha University, Kyoto 610-0321, Japan
[3]POLOPT Technologies GmbH Bruggerstrasse 44, 5400 Baden, Switzerland

This paper proposes novel accuracy improvement and high speeding techniques for double layer charge (DLC) formulation in shielding problems. Although the DLC formulation has advantages over the direct methods from the viewpoint of computational cost, the calculation accuracy can be worse because of cancellation errors. In order to improve the accuracy, we apply a new technique based on difference field concept to the DLC formulation. In addition, we succeed in shortening a calculation time by simplifying the DLC formulation. Numerical results which verify the effectiveness of the proposed methods are presented.

Index Terms— **Difference magnetic field, double layer charge, integral equations, magnetic shielding, magnetostatics.**

I. Introduction

High aspect ratio models such as magnetic shields are not easy to analyze accurately by the finite element method. On the other hand, they can be analyzed accurately by the direct boundary element method (BEM) although requiring high computational costs. Therefore the use of indirect BEMs, which contain almost half the number of unknown variables compared with the direct BEM, is suitable for shielding problems. In particular, the double layer charge (DLC) formulation is one of the most effective methods because its accuracy is better than other indirect BEMs in most cases [1]. However, the computational accuracy of the magnetic field in the shielding space calculated by the indirect BEMs is not good because of cancellation errors regardless of their shapes and the use of the Galerkin or collocation method [2]. It was reported that the accuracy of the DLC formulation can be improved by using the direct BEM as the postprocessing [2]-[3] although using two types of BEMs is complicated. So in this paper, we apply the difference field concept which is used in the single layer formulation [4]-[6] to the DLC formulation.

We also halve the number of unknowns of the DLC formulation by utilizing a characteristic of shielding model without losing the accuracy.

II. Numerical Methods

A. Double Layer Charge Formulation

The DLC formulation for linear media [1]-[3] is given as

$$\frac{(\mu^*-1)\Omega(\boldsymbol{r})+4\pi}{4\pi(\mu^*-1)}\sigma_d(\boldsymbol{r})+\int_S \sigma_d(\boldsymbol{r}')\frac{(\boldsymbol{r}-\boldsymbol{r}')\cdot\boldsymbol{n}(\boldsymbol{r}')}{4\pi|\boldsymbol{r}-\boldsymbol{r}'|^3}\mathrm{dS}'=-\mu_0\varphi_{He}(\boldsymbol{r}), \quad (1)$$

where σ_d is the magnetic double layer charge, φ_{He} is the potential made by exciting current, μ_0 is the permeability of free space, $\boldsymbol{r}$ and $\boldsymbol{r}'$ denote observation and integration positions, respectively. The relative ratio μ^* is defined as $\mu^* = \mu_{ri} / \mu_{ro}$, where μ_{ri} and μ_{ro} are the permeability of inner and outer region of magnetic material, respectively in case of shielding model. $\Omega(\boldsymbol{r})$ is the solid angle subtended by the inner region at $\boldsymbol{r}$ and $\boldsymbol{n}$ is the unit outward normal vector from the inner region with μ_{ri} to the outer region with μ_{ro}. σ_d is approximated by a linear function. Equation (1) is discretized by using a collocation approach with a triangular surface mesh.

The magnetic flux density $\boldsymbol{B}$ at any point is given as

$$\boldsymbol{B}(\boldsymbol{r})=\boldsymbol{B}_{\sigma d}(\boldsymbol{r})+\boldsymbol{B}_e(\boldsymbol{r}), \quad (2)$$

where $\boldsymbol{B}_e$ is exciting magnetic flux density and $\boldsymbol{B}_{\sigma d}$ is the magnetic flux density produced by σ_d.

B. Difference Field Concept for DLC formulation in Shielding model

For simplicity, we discuss single layer magnetic shields as an example. Generally, the direction of $\boldsymbol{B}_e$ is reverse to that of $\boldsymbol{B}_{\sigma d}$ in (2) in the shielding space. As μ^* increases, $\boldsymbol{B}$ inside the shielding space approaches 0. In other words, the order of the magnitude of $\boldsymbol{B}$ becomes quite different from that of $\boldsymbol{B}_e$ and $\boldsymbol{B}_{\sigma d}$ in (2), and the cancellation errors become larger. In the following, we explain how to avoid large cancellation errors by using the difference field concept.

When $\mu^*=\infty$, (1) is represented as

$$\frac{\Omega(\boldsymbol{r})}{4\pi}\sigma_d^{\infty}(\boldsymbol{r})+\int_S \sigma_d^{\infty}(\boldsymbol{r}')\frac{(\boldsymbol{r}-\boldsymbol{r}')\cdot\boldsymbol{n}(\boldsymbol{r}')}{4\pi|\boldsymbol{r}-\boldsymbol{r}'|^3}\mathrm{dS}'=-\mu_0\varphi_{He}(\boldsymbol{r}). \quad (3)$$

Here, we define $\delta\sigma_d$ as $\delta\sigma_d = \sigma_d - \sigma_d^{\infty}$. Substituting the left-hand side of (3) into the right-hand side of (1), and simplifying the equation, we obtain

$$\frac{(\mu^*-1)\Omega(\boldsymbol{r})+4\pi}{4\pi(\mu^*-1)}\delta\sigma_d(\boldsymbol{r})+\int_S \delta\sigma_d(\boldsymbol{r}')\frac{(\boldsymbol{r}-\boldsymbol{r}')\cdot\boldsymbol{n}(\boldsymbol{r}')}{4\pi|\boldsymbol{r}-\boldsymbol{r}'|^3}\mathrm{dS}'=\left(\frac{-1}{\mu^*-1}\right)\sigma_d^{\infty}(\boldsymbol{r}). \quad (4)$$

When $\mu^*=\infty$ $\boldsymbol{B}$ inside the shielding space is 0 and from (2) $\boldsymbol{B}^{\infty}_{\sigma d}+\boldsymbol{B}_e=0$ with $\boldsymbol{B}^{\infty}_{\sigma d}$ produced by σ_d^{∞}. Thus the magnetic flux density inside the shielding space $\boldsymbol{B}_{\mathrm{in}}$ is represented as

$$\begin{aligned}\boldsymbol{B}_{in} &= \boldsymbol{B}_{\sigma d}+\boldsymbol{B}_e=\boldsymbol{B}_{\sigma d}-\boldsymbol{B}^{\infty}_{\sigma d}=\boldsymbol{B}_{\sigma d\_out}+\boldsymbol{B}_{\sigma d\_in}-\boldsymbol{B}^{\infty}_{\sigma d\_out}-\boldsymbol{B}^{\infty}_{\sigma d\_in}\\ &=\left(\boldsymbol{B}_{\sigma d\_out}-\boldsymbol{B}^{\infty}_{\sigma d\_out}\right)+\left(\boldsymbol{B}_{\sigma d\_in}-\boldsymbol{B}^{\infty}_{\sigma d\_in}\right)=\boldsymbol{B}_{\delta\sigma d\_out}+\boldsymbol{B}_{\delta\sigma d\_in},\end{aligned} \quad (5)$$

where $\boldsymbol{B}_{\sigma d\_out}$ and $\boldsymbol{B}_{\sigma d\_in}$ denote $\boldsymbol{B}$ produced by σ_d on the outer and inner boundaries of the shielding material, respectively, and also $\boldsymbol{B}_{\delta\sigma d\_out}$ and $\boldsymbol{B}_{\delta\sigma d\_in}$ denote $\boldsymbol{B}$ produced by $\delta\sigma_d$ on the outer and inner boundaries, respectively. When μ^* is large, $\boldsymbol{B}_{\sigma d\_out}$ and $\boldsymbol{B}^{\infty}_{\sigma d\_out}$ are almost the same. Therefore, the order of the magnitude of $\boldsymbol{B}_{\delta\sigma d\_out}$ is much smaller than that of $\boldsymbol{B}_{\sigma d\_out}$. The same discussion holds true for $\boldsymbol{B}_{\delta\sigma d\_in}$. Thus as indicated in (5), the large cancellation errors can be avoided.

C. Simplification Technique for DLC Formulation

When the shielding materials are really thin and μ^* is large, the difference of the magnetic scalar potentials on the outer and inner boundaries is obviously small. Then distribution of σ_d becomes almost the same on the outer and inner boundaries. In this case, the difference of σ_d is also very small. Utilizing this feature the number of unknowns of the DLC formulation can be halved as shown in Fig. 1.

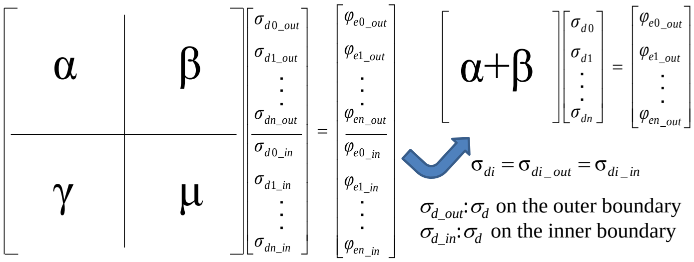

Fig. 1. Simplifying the matrix of the DLC formulation..

III. Numerical Results and Discussions

To confirm the effectiveness of the proposed method, we analyze a spherical shell model in a uniform magnetic field of 100 A/m, where the inner radius a_1 is 4.9 cm and outer radius a_2 is 5.0 cm. We evaluate the magnetic field $\boldsymbol{H}$ using the direct BEM-based postprocessing, the difference field concept and their simplified versions. Then we compare the computed values H_c with theoretical ones H_t and show relative errors in Fig.2, where the relative error is defined as $|H_c\text{-}H_t|/H_t$. In addition, to investigate the applicable scope of the simplification technique, we vary a_1 from 2.5 cm to 4.9 cm when μ^*=1000. The results are shown in Fig. 3. Figs. 2 and 3 show the average of the relative errors of $\boldsymbol{H}$ inside the inner boundary (i.e., shielding space). The rectangle and triangle plots denote the results by the DLC formulation with the direct BEM-based postprocessing and the DLC formulation based on the difference field concept, respectively. The open symbols correspond to the results of their simplified methods. We divided the surface of the single layer shield into 2400 elements and 1204 nodes. The number of unknowns of each method is shown in Table I. In Table I, the 1st step stands for solving (1) or (3) and the 2nd step stands for solving the direct BEM formulation or (4).

TABLE I
Number of Unknowns of Each method

	DLC_post-processing	simplified DLC_post-processing	DLC_difference field	DLC_simplified difference field
1st step	1204	602	1204	602
2nd step	602	602	1204	602

In Fig. 2, when μ^*> 100, the large cancellation errors occur in the DLC formulation. We can avoid those errors by using the difference field concept. On the other hand, the accuracy by the difference field concept is a little worse than the direct BEM-based postprocessing because of the small cancellation errors. We can also confirm that the number of unknowns of the DLC formulation can be halved with keeping the adequate accuracy.

As seen in Fig. 3, the simplification technique is effective when the thickness of the shielding material is within about 1/10 of the outer radius in this numerical example.

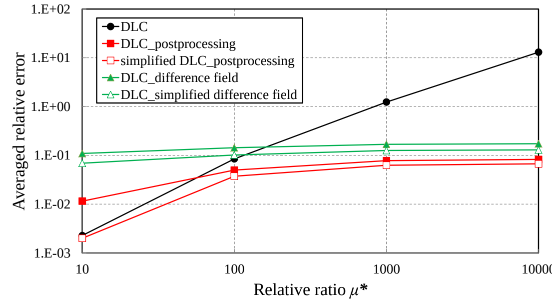

Fig. 2. Averaged relative error of computational results versus μ^*.

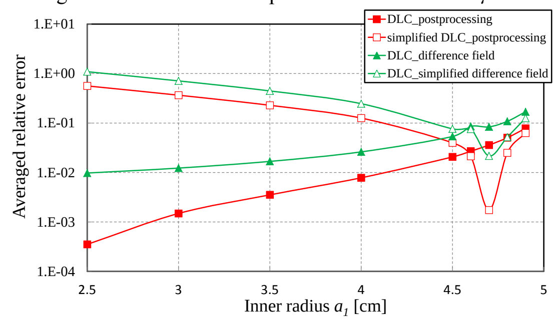

Fig. 3. Averaged relative error of computational results when a_1 varies from 2.5 cm to 4.9 cm.

IV. Conclusion

Our results indicate we can avoid the large cancellation errors in shielding problems by applying the difference field concept to the DLC formulation. Also, we can halve the number of unknowns of the DLC formulation with keeping the adequate accuracy in the case of thin shielding models. From the view point of the computational costs, the use of the difference field concept with simplification technique is very effective when the thickness of the shielding material is sufficiently thin.

In the fullpaper, we will investigate the scope of the simplification tachnique in more detail. We will also analyze more practical shielding problems.

V. References

[1] Z. Andjelic, G. Of, O. Steinbach and P. Urthaler, “Boundary element methods for magnetostatic field problems: a critical view,” Computing and Visualization in Science, vol. 14, iss. 3, pp. 117-130, Mar. 2011.

[2] T. Yoshioka, S. Wakao, Y.Takahashi, L. Kielhorn, K. Ishibashi and K. Fujiwara, “Accuracy Improvement of Boundary Element Methods for Magnetostatic Field Problems,” *The Papers of Joint Technical Meeting*, *IEE Japan*, SA-14-003, RA-14-003, Jan. 2014 (in Japanese).

[3] K. Ishibashi, T. Yoshioka, S. Wakao, Y.Takahashi, Z. Andjelic and K. Fujiwara, “Improvement of unified boundary integral equation method in magnetostatic shielding analysis”, *IEEE Trans. Magn*., vol. 50, no. 2, 7002404, Feb. 2014.

[4] I. D. Mayergoyz, M. V. K. Chari and J. D’angelo, “A new scalar potential formulation for three-dimensional magnetostatic problems”, *IEEE Trans. Magn*., vol. MAG-23, no. 6, pp. 3889-3894, Nov. 1987.

[5] S. Babic, C. Akyel, and M. M. Gavrilovic, “Calculation improvement of 3D linear magnetostatic field based on fictitious magnetic surface Charge,” *IEEE Trans. Magn*., vol. 36, no. 5, pp. 3125-3127, Sep. 2000.

[6] D. H. Kim, I. H. Park, M. C. Park and H. B. Lee, “3-D magnetostatic field calculation by a single layer boundary integral equation method using a difference field concept,” *IEEE Trans. Magn*., vol. 36, no. 5, pp. 3134-3136, Sep. 2000.